Metamodel Optimization of a Complex, Rural-Urban Emergency Medical Services System

Matthew Snyder[a], Byran J. Smucker[a]

[a] *Department of Statistics, Miami University, Oxford, Ohio,*

Abstract

Complex simulation systems, such as those involving emergency medical services (EMS), are computationally difficult to optimize. Metamodeling is an attractive alternative, in which a sample of system configurations is evaluated using simulation, and a fast predictive model is developed as a surrogate for the slow simulator. Though the metamodeling literature is extensive, there has been little exploration of how much data is required to effectively optimize the underlying simulation system with a metamodel. In this work, we study the EMS system in northern St. Louis County, Minnesota, with the goal of finding improved station configurations. The underlying physical system is complex, with 12 stations spread across both rural and urban areas and a fairly large geographic footprint. We use a decade of call data from this system to develop and validate a stochastic discrete event simulator (DES), and then use the simulator and raw data to select realistic station configurations to train the metamodel. We provide results first for just a single station within the system, and then examine increasingly complex settings culminating with consideration of all 12 stations. Overall, we find that though the metamodeling approach was effective for simpler cases, it requires a tremendous amount of data for the complex settings. Specifically for this setting, we found improved configurations for the one- and two-station cases, but the amount of data required to produce effective metamodels for the five- and twelve-station versions of the system was computationally infeasible given our current DES and optimization implementations.

Keywords: Emergency medical services, Discrete event simulation, Metamodeling, Particle swarm optimization, Random forest

1. Introduction

Emergency Medical Services (EMS) departments are tasked with responding quickly to the medical needs of people in a community. The speed at which they can respond depends on the available resources, including ambulances and staff, as well as the location of the stations with respect to the calls [1, 2]. It has been shown that adjusting the station locations or available equipment from a finite list of candidate configurations can reduce response times and save lives [3, 4, 5, 6]. Due to the complex and interconnected nature of these systems, simulations can be developed to test alternative resource allocation strategies [7, 8, 9]. In particular, discrete event simulation (DES) has been shown to accurately model EMS [10, 11, 12, 13] and other healthcare systems [14, 15, 16, 17, 18] due to its flexibility and ability to handle constrained resources. These simulators can then be used to compare how various system configurations affect response times [19, 20, 21, 22, 23].

Simulations, including DES, are computationally intensive, which becomes a limiting issue as the number of configurations under consideration increases. Optimizing a simulation over an entire region requires many evaluations, making the procedure computationally intractable. Thus, this paper seeks to develop a simplified predictive model, called a metamodel, based on many runs of the DES. Because metamodels only consider the relationship between the inputs and outputs of the simulation, rather than accounting for each of the complex aspects of the simulation, they can be optimized much more quickly [24]. Specifically, we study sampling techniques and the amount of training data needed to fit a metamodel well enough to be usefully optimized. This is our main contribution. We are motivated by extensive work on a real EMS system in northern St. Louis County, MN, USA, as well as by a company doing analytics and optimization work for EMS systems, and use extensive call data from the system to develop a DES, and then sample many station configuration, fit a metamodel, and optimize. As discussed in Section 2, there is little in the literature that addresses the nexus of issues that we do: optimization using a metamodel built to mimic a DES which in turn is modeling an underlying complex EMS system. To our knowledge, this has not been done in an EMS simulation setting, and rarely in the metamodeling literature more broadly.

The following section presents a review of related simulation and metamodeling work, Section 3 describes the methodology of our studies, and Section 4 contains the results, followed by a discussion of the findings and areas for future work.

2. Related Work

Many simulation and optimization models have been developed in the study of EMS systems. Li et al. [25] presents a review of several mathematical models that have been used in station location planning and optimization. The authors also present an overview of simulation techniques for the location and allocation of facilities, since simulation is more flexible for studying such complex systems. Though they discuss testing a number of policies for ambulance allocation and deployment and comparing multiple pre-defined ambulance location configurations, they do not consider a full optimization of station locations. Aringhieri et al. [26] provides a review of EMS ambulance location, relocation, and dispatching policy problems, including the use of simulation models and specifically DES. Of particular interest to our work, the study by Mason [27] explores a simulation-optimization of vehicle base locations, but only performs a local optimization by perturbing existing station locations. As seen in these papers, simulation allows more flexibility and accuracy in modeling EMS systems, but comes with complexity and computational expense. This has limited the optimization of station locations to selecting from pre-defined candidate locations or implementing a local optimization procedure. The use of metamodels as a surrogate for the simulation model is a technique that can simplify the underlying structure while maintaining high accuracy to enable a full optimization.

In recent years, metamodeling has been widely studied. For EMS systems, metamodels have been used to compare and optimize both response times and survival rates by changing ambulance locations [28] and dispatch policies [29]; these models were based on agent-based simulators. More generally, comparisons of different types of metamodels, such as regression splines, kriging, artificial neural networks, and random forests, as well as different sampling approaches, including space-filling designs and adaptive sampling, have been conducted in many contexts on a variety of simulators [30, 31, 32, 33, 34, 35], including discrete event simulators [36, 37]. Depending on the context of the problem and the ultimate goal, the recommended model and approach varies. For simulators of EMS systems, Hopkins and Smucker [38] found that k-nearest neighbors and random forest models yield the highest accuracy and predictive power.

The goal in metamodeling is to create a highly accurate model using the least amount of training data, since this training data is expensive to obtain. Jin et al. [39] was an early researcher of metamodel performance using different sample sizes and problem complexities. For large scale problems with at least 10 predictors, the authors compare the accuracy of three sample sizes – scarce, small, and large – using R^2, relative average absolute error (RAAE), and relative maximum absolute error (RMAE). The samples are generated using Latin hypercubes, with sizes determined as functions of the number of parameters p, ranging from $3p$ to $3(p+1)(p+2)/2$. They found that the accuracy tended to increase as sample size increased, with average R^2 values near 70 for the large scale, nonlinear problems. Yang et al. [40] compared the RMSE of five types of metamodels fit using training sets with sizes ranging from $3p$ to $36p$, where $p = 4$, for a complex, nonlinear finite element model. Similarly, Kim et al. [41] studied the accuracy of metamodels built on samples of $3p$, $5p$, and $7p$ points using RMSE for sample mathematical problems with 2 through 8 predictor variables. Kianifar and Campean [42] provided a recent, comprehensive literature review of metamodeling techniques with the goal of creating a guide for engineering professionals; the authors compared many facets of metamodeling for several mathematical-based engineering problems, including two different samples sizes (10- and 30-times the number of predictors), using a normalized RMSE. The general consensus for these studies was that increasing the sample size resulted in higher accuracies, but at varying rates for different model types, problem complexities, and error types.

Other studies have compared the accuracy of simulation-based metamodels built on several pre-defined sample sizes, typically ranging from several dozen to several hundred training points, and occasionally reaching a thousand points [43, 44, 45, 46, 47, 48]. One exception is Ding and Zhang [49], who explored large-scale simulation metamodeling in settings with 10, 20, 50, and 16,675 predictors, and tested sample sizes ranging from 200 points to 30,000 points. They compared the RMSE at increasing sample sizes using multiple sampling designs, and generally found sharp decreases in errors that eventually plateaued; however, the authors focused on metamodeling for simulation prediction, rather than optimization where alternative methods may be more efficient.

Once constructed, metamodels can be used to optimize the underlying simulations, as in Osorio and Chong [44] who optimize signal plans in simulation-based transportation systems, and Ju et al. [46], who optimize turbomachinery designs in Monte Carlo simulation problems. Zeinali et al. [50] used a metamodeling approach to optimize a DES for emergency departments, and discovered a resource configuration that reduced waiting times by 48% for a particular hospital. The metamodel was fit using fewer than 100 points, and the solution was found quickly and was near-optimal.

Unlike the existing literature, our research seeks to optimize the location of the EMS stations over a continuous region rather than over a finite list of candidate locations or a perturbation of the existing system. To accomplish this, we fit metamodels over the underlying simulation, using as input only the locations of the EMS stations. We wish to better understand how much training data is required to obtain metamodels reliable enough to optimize. Due to the intrinsic, interconnected nature of the rural-urban EMS system we study, our problem is significantly more complicated than the studies in the current literature and requires substantially more data. Further, the traditional, commonly-used space-filling designs are not practical in this setting since they waste computational resources on illogical station configurations, so alternative sampling methods are considered.

3. Data and Methods

In order to evaluate the efficacy of this metamodeling approach and determine the amount of data needed to obtain useful results, we examine four versions of a case study, each of increasing complexity. These include a simple setting with one station, a two-station and five-station setting that account for the interaction of stations, and a full twelve-station setting to model at least a simplified version of the entire EMS system of northern St. Louis County, Minnesota. For each situation, we created a DES and generated several samples of station configurations to run through the DES. This resulted in a simulated 90th percentile for each configuration that was then used as the response for the fitted metamodel. Once fit, the metamodels were optimized and the proposed configurations were validated on the DES. If improvements in the metamodel and proposed configurations could be expected, the process repeated with a larger sample size, as illustrated by Figure 1.

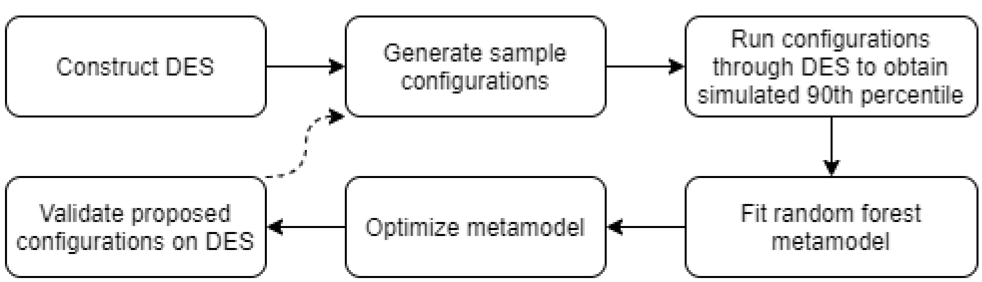


Figure 1: Flowchart of the methodology used in this article.

All data cleaning, simulation, model fitting, and optimization was done in R [51]. The packages `tidyverse` [52], `simmer` [53], `osrm` [54], `caret` [55], `randomForest` [56], and `pso` [57] were used extensively.

3.1. Data Handling

The data for this study was provided by Allen Lewis, Fire Chief and Emergency Manager of the Virginia Fire Department in Virginia, Minnesota. Calls related to medical emergencies between 2009 and 2019 were included, extracted from two sets of data. Calls collected between December 27, 2009 and April 3, 2016 were reported weekly, while those collected between January 1, 2018 and December 30, 2019 were reported daily. For every call, the date (or week), type of emergency, radio name of the responding vehicle, address, latitude, and longitude were recorded, along with the time that the call arrived as well as the times that the vehicle was dispatched, enroute, on scene, and cleared from the emergency.

We constructed several additional variables based on the data. Both the station and vehicle type of the responding vehicle were extracted from the radio name; ambulances, Medical Response vehicles, and Battalion Chief vehicles were classified as "EMS", while other vehicles were recorded as "Fire". A binary city variable marked calls that were located in the cities of Virginia, Mountain Iron, Hibbing, Ely, Eveleth, or Chisholm and within 5 miles (8.05 km) of the station as true, while others were marked false. The time of day was recorded as "early morning" for calls arriving before 6:00 a.m., "morning" for calls arriving before noon, "afternoon" for calls arriving before 6:00 p.m., and "evening" for other times. The season was recorded as "winter" for calls arriving from December to February, "spring" for those arriving from March to May, "summer" for those arriving from June to August, and "fall" for those arriving from September to November.

To obtain accurate and reliable data for this study, we implemented several data cleaning procedures. In all, three datasets were constructed from the raw data. The first dataset, denoted A, omits calls with missing locations and missing or illogical times. It also omits calls whose times were judged to be unreasonable, when compared to the OpenStreetMap time. Dataset A is the most filtered set of calls (Table 1) and is used whenever it is necessary to have a set of calls with reliable times. More details regarding Dataset A are provided in Supplementary Materials Section 1. Another dataset, denoted B, was generated to assess the call frequency over time by computing the difference in call arrival times between unique emergencies. Only the 2018 and 2019 call data was used, as earlier data was reported weekly and not daily. Time of day and season variables were also computed as described above based on the arrival times. Finally, dataset C consists of call locations from the entire decade of call data. A binary city variable marked calls that were located in any city (Virginia, Mountain Iron, Hibbing, Ely, Eveleth, or Chisholm) as true, while others were marked false.

Together, these three datasets served as the foundation for the simulation and analysis of each version of the case study. Based on the number of stations in each scenario, we filtered the datasets to include a subset of the responding stations. The one-station setting only accounted for the Virginia station; the two-station version added the Eveleth station; the five-station version added the Hibbing, Buhl, and Chisolm stations; and the twelve-station setting included the full system. Table 1 shows the amount of data in each dataset for each scenario based on our data handling. We then used these calls and locations as the basis for the discrete event simulator described next.

Scenario	Observations in Dataset			
	Dataset A	Dataset B	Dataset C	
	(Calls)	(Calls)	(Calls)	(Unique Locations)
One Station	6,240	8,788	22,335	5,285
Two Station	7,129	10,333	27,877	6,461
Five Station	16,524	18,768	49,640	11,588
Twelve Station	19,625	23,233	64,847	15,956

Table 1: Number of calls used for each dataset in each case study. Dataset A is filtered in order to have calls with reliable response times. Dataset B is a set of calls used to measure frequency and time between calls. Dataset C is a complete set of call locations.

3.2. Discrete Event Simulation

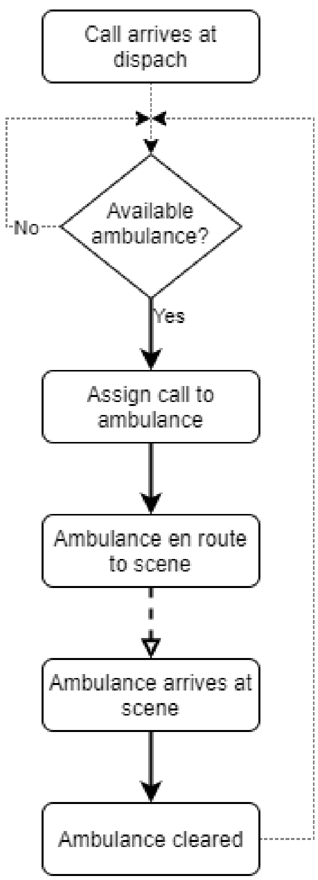


Figure 2: Discrete Event Simulation process. The various aspects of the response time are denoted by solid arrows. The travel time is denoted by the dashed arrow.

As described in the previous section, discrete event simulation (DES) is common in the EMS space. Thus, to model the EMS system, we created a DES using the `simmer` package [53] which corresponds to the first step in Figure 1. DES is a method of modeling complex systems comprised of events in time, in which the system remains unchanged between events. In an EMS system, these events correspond to the call arriving to the dispatcher, the dispatcher assigning an ambulance, the ambulance leaving for the scene, arriving at the scene, and being cleared from the emergency, as seen in Figure 2. The coordinates of the stations are the input to the DES, and the ambulances are the constrained resources. For this analysis, we assumed that stations have either two or four ambulances based on the current resources in Northern St. Louis County. Once a call arrives and is assigned to a station, an ambulance is seized until the call is cleared. If no ambulances are available, the call is queued and waits until a vehicle is cleared for the one-station case, or is assigned to the next closest station for the more complex cases. The location of the call is randomly selected from a list of all locations found in dataset C.

The time between call arrivals, dispatch to assignment times, assignment to enroute times, and onscene to clear times are randomly generated from gamma distributions and are represented by the solid arrows in Figure 2. Based on the data, we investigated whether these distributions changed appreciably as a function of variables like time of day and time of year. Using these findings, we adjusted the distribution for time between call arrivals based on time of day and season, assignment to enroute times based on time of day and vehicle type, and the dispatch to enroute and on scene to clear times based upon the binary city variable and vehicle type. As an example, the parameter values for the one-station setting are provided in Table 2. The remaining time needed to build the DES is the travel time, represented by the dashed arrow in Figure 2. We estimated this travel time using a linear regression model that calibrated the OpenStreetMap times to realistic emergency response travel times, while accounting for variables like time of day and season. Based on a prediction interval from this regression analysis, we generated a distribution from which the travel time was drawn. Further details are provided in the Supplementary Material Section 2.

Putting all these fitted distributions together, we constructed the DES and ran it for one simulated year. The response time for each generated call was calculated as the difference in time between the call being assigned to a station and the ambulance arriving at the scene. Then, for a given configuration, we computed the 90th percentile of the response times across all calls since this is the primary metric of interest. In Section 3 of the Supplementary Material we compare the simulated 90th percentiles to the 90th percentiles based on the historical data. Overall, the simulation provides response time distributions that are very similar to those found in the data.

Time Difference	**Time of Day**	**Early Morning**		**Morning**		**Afternoon**		**Evening**	
	Season	**Shape**	**Rate**	**Shape**	**Rate**	**Shape**	**Rate**	**Shape**	**Rate**
Call Arrival	**Spring**	1.271	0.007	0.768	0.005	0.889	0.010	1.106	0.011
	Summer	1.501	0.008	0.842	0.005	1.043	0.012	1.180	0.012
	Fall	1.234	0.007	0.745	0.005	0.972	0.011	1.208	0.013
	Winter	1.428	0.008	0.774	0.005	1.055	0.013	1.243	0.013
Assignment to Enroute	**Vehicle - EMS**	1.572	0.817	0.824	0.468	0.952	0.868	1.191	0.943
	City Indicator	**In City**				**Not in City**			
	Vehicle Type	**Shape**		**Rate**		**Shape**		**Rate**	
Dispatch to Assignment	**EMS**	2.578		1.108		1.992		0.677	
Onscene to Clear	**EMS**	1.789		0.031		1.617		0.022	

Table 2: Parameter values for the gamma distributions used in the discrete-event simulator.

3.3. Metamodeling

Once constructed, we ran the DES many times with different station configurations to determine the 90th percentile of the response times under these different conditions, as illustrated in the second and third steps of Figure 1. Using a set of station locations as inputs and simulated 90th percentiles as outputs, we then fit a random forest metamodel. Since the DES takes time to run, the goal is to fit the metamodel on the smallest sample of station locations that still yields accurate and informative results. Table 3 shows all sample sizes considered for the four settings.

Scenario	Sample Sizes
One Station	50, 100, 200, 300, 400, 500, 1000
Two Station	500, 1000, 2000, 3000, 4000, 5000
Five Station	2000, 10000, 50000
Twelve Station	5000, 10000, 50000

Table 3: Sample sizes studied for each version of the case study.

Intuitively, a well-situated configuration that reduces the 90th percentile of response times would have stations located in the vicinity of the majority of calls, yet still spread throughout the region. Thus, rather than using a space-filling design to generate sample locations, we used a weighted sampling technique based on the spatial density of calls. This allows for more sample data in the intuitive locations, increasing the precision of the results in these areas. We computed the spatial density of the calls in dataset C using the `density.ppp` function of the `spatstat` R package [58], which increased the probability of choosing station locations in areas that have many emergency calls and decreased the probability of choosing irrelevant station locations. Details of the density estimation are provided in Section 4 of the Supplementary Material.

The one- and two-station versions simply selected one or two locations from this density for each configuration to generate the samples. For the five- and twelve-station versions, we added an additional constraint that forced all locations in a configuration to be at least four miles (6.44 km) apart. This constraint was implemented due to the extremely high density of calls in the city of Virginia, which resulted in many configurations with several stations in very close proximity. We selected a distance of four miles since the distance between the two closest stations in the current configuration, as well as the distances between proposed stations in the two-station setting were all just over four miles (6.44 km) apart. While this added constraint introduces some limitations in the fitted metamodels, it allows for more exploration of intuitive configurations, those with stations spread throughout the region, while requiring less data. A brief exploratory study comparing several alternative sampling techniques is provided in Section 5 of the Supplementary Material, which found that using the constrained weighted technique ultimately resulted in the most promising proposed station configurations.

Once the sample of input configurations was generated, we fit the metamodel, corresponding to the fourth step of Figure 1. For this analysis, we chose random forest models due to their high predictive power in EMS settings [38] and fit a model for each of the sample sizes considered. Details on the implementation of the random forest are provided in Section 6 of the Supplementary Material.

3.4. Optimization

We then optimized the fitted metamodels using standard particle swarm optimization, specifically SPSO 2007, in order to find the configuration that the metamodel predicts will have the shortest 90th percentile response time. Corresponding to the fifth step in Figure 1, this technique is based on a swarm of several configurations, called particles. The metamodel is evaluated for each particle, and each particle is assigned a velocity. The particles then move around the search space until the global optimum is found. In particular, the optimization problem can be specified as follows:

$$\begin{aligned} &\text{Minimize } \; f(lat_1, long_1, lat_2, long_2, \ldots, lat_K, long_K) \\ &\quad s.t. \;\; (lat_i, long_i) \in (\min_{n \in \mathcal{C}} y_n, \max_{n \in \mathcal{C}} y_n) \times (\min_{n \in \mathcal{C}} x_n, \max_{n \in \mathcal{C}} x_n), \; i = 1, 2, \ldots, K \end{aligned} \tag{1}$$

where f is the metamodel-predicted 90th percentile of the response times for a configuration with K stations, $K = 1, 2, 5, 12$; station k is located at latitude lat_k and longitude $long_k$; x_n and y_n are the longitude and latitude of call n in dataset C; and $\mathcal{C}$ is the set of all row indices in dataset C. Note that while the data used to fit the models included a distance constraint between stations for the five- and twelve-station cases, the optimization procedure did not, allowing stations to become close or even overlap. Given the highly nonlinear and black-box nature of f, optimization in this setting is limited to heuristics and we chose to use particle swarm. PSO has been shown to find global optimal solutions quickly [59, 60, 61], including in healthcare and ambulance location models [62, 63].

For this study, we performed the optimization procedure on the metamodel 100 times using different initial values. These initializations include the current station configuration as well as 99 other configurations sampled from the call densities, using the same method described in Section 3.3. Each algorithm try terminated after 50 iterations passed without any improvement in the objective function. This technique provides a set of station locations that have low metamodel-predicted 90th percentile response times. Rather than proposing a single solution, it provides a set of potential configurations allowing for additional exploration and visualization of trends, which is desired since the optimization of metamodels has been shown to find improved, but not necessarily optimal, solutions for the underlying system.

3.5. Validation

As the final step in Figure 1, we tested each location proposed by the 100 optimizations of the metamodel (Section 3.4) on the DES and compared them to the current configuration. Due to the inherent randomness in the simulator, we ran the configurations several times and calculated the average 90th percentile response time. To start, the DES was run 100 times for the current station location and the mean and standard deviation of the 90th percentile response times were computed. Next, we conducted a simple statistical power analysis to determine how many runs of the DES were needed in order to detect differences of at least 30 seconds compared to the mean 90th percentile of the current station location, with 90% power and a significance level of 0.05. The DES was then run for each of the station configurations proposed by the optimization procedure, and average 90th percentile response times and corresponding 95% confidence intervals were compared to those from the current system.

Note that since the metamodels for the five- and twelve-station cases were substantially more complex than the simpler cases, a majority of the 100 optimization procedures resulted in different configurations.

Further, these simulations took additional time to run, so we tested each proposed configuration on the DES only once to start. Then, based on this single run, the configurations with a simulated 90th percentile below the upper 95% confidence interval bound for the current configuration were run on the DES additional times using the findings from the sample size analysis. Finally, we computed the means and 95% confidence intervals of the simulated 90th percentile of response times for these proposed configurations and compared them to the current system.

4. Results

Using the methodology described in Section 3, we present the results for our study of metamodeling as a strategy to perform optimization in a complex simulation environment. We report on each of the one-, two-, five- and twelve-station scenarios in order to understand the metamodeling data needs in increasingly complex environments.

4.1. One Station

In order to fit a random forest metamodel over the DES, we generated weighted samples of seven incremental sizes as inputs, as described in Section 3.3. Figure 3 shows the samples of size 50, 200, 500, and 1,000 in blue on top of all possible call locations shown in black. For samples of 200 and fewer locations, the stations were primarily located in the center of the region, while increasing the sample sizes to 500 or 1,000 expanded the coverage to the outlying areas as well. Using these locations as inputs, we then ran the DES at each location, computed the corresponding 90th percentile response time, and fit a metamodel to each simulated dataset using the station locations as inputs and the 90th percentile response times as outputs. Figure 4a shows the goodness of fit of the models by sample size using the cross-validated R^2 of the selected random forest metamodel along with standard error bars. Although the R^2 is relatively high for a sample size of 50, it drops to a mean of 64.1% for a sample size of 100. It then approaches 80% for samples of size 200 and 300 before peaking with values around 88% for the larger sample sizes. The variability in the R^2 values is smallest for samples of size 400 and above.

Next we optimized each of the seven metamodels using the particle swarm optimization procedure and 100 different initializations, which resulted in a set of proposed station locations. Each of these locations with a unique 90th percentile based on the metamodel, as well as the current location, were then used as the input to the DES. Based on the sample size analysis (see Section 3.5), ten runs of the DES were required to detect mean 90th percentile differences of at least 30 seconds below that of the current station. We computed the mean and 95% confidence intervals for the simulated 90th percentile response time across these ten runs, which are shown in Figure 4b. Each bar corresponds to a unique location proposed by the optimization procedure, and the size of each dot corresponds to the proportion of the 100 procedures that resulted in a location with the same 90th percentile response time based on the metamodel.

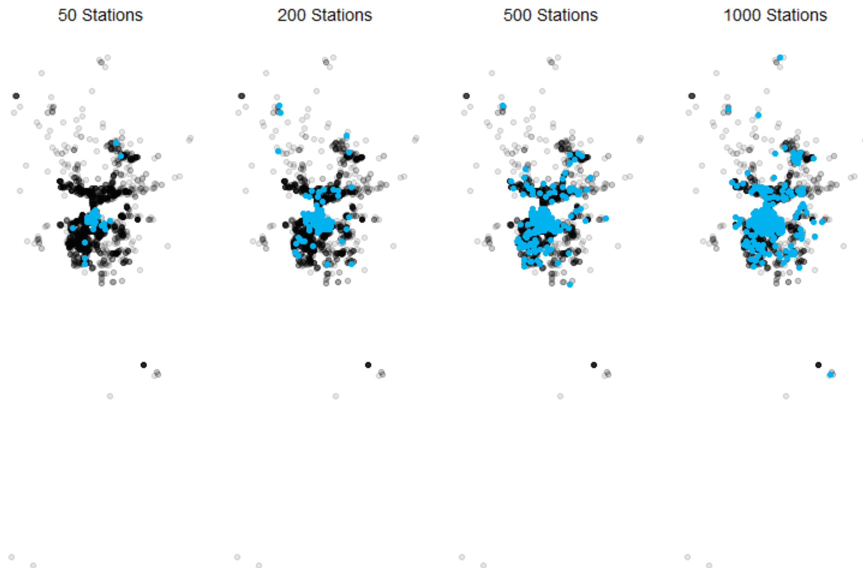
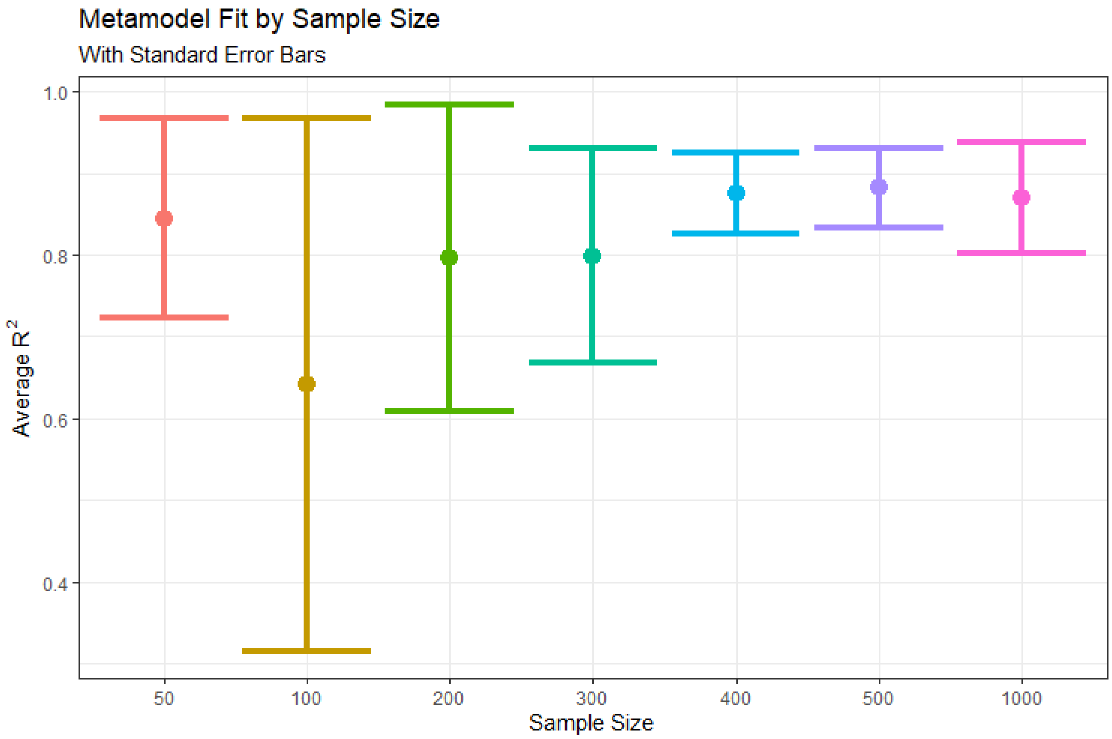


Figure 3: Sample locations for the one-station setting, used as inputs for the metamodels.

Based on this plot, the proposed station locations from the optimization of metamodels built on all considered sample sizes reduced the simulated 90th percentile response time by at least 30 seconds compared to the current station. Note that for the sample size of 50, 1 optimization run resulted in a location with an average 90th percentiles response time of 142 minutes which is not included in Figure 4b. The best location, on average, was found by one optimization procedure with a sample size of 200, for a mean 90th percentile response time of 17.27 minutes compared to the current station's mean of 19.07 minutes. The next best location was found by 80% of the optimization procedures on the metamodel built on 1,000 locations, with a mean 90th percentile response time of 17.43 minutes.

In addition to the 90th percentile, Figure 5a shows the distribution of response times for several of these proposed locations as well as the current location from one run of the DES. While all distributions are right skewed, the current station's response times distribution has a peak near 3.75 minutes. The other curves, shown for sample sizes of 100, 400, and 1,000, have peaks between 4.5 and 6.5 minutes minutes. This indicates that although the proposed locations reduce the simulated 90th percentile of response times, effectively reducing the amount of time needed to reach calls in outlying areas, they require additional time for many of the calls that are within five minutes of a response under the current system.

To understand the results described above, Figure 6a plots several of the proposed locations in relation to the city of Virginia. The current station, shown in red, is in the center of the city, allowing for a fast response to many calls in a dense area. However, the proposed locations based on the metamodel optimization are on the western side of the city, along US 53 or US 169, which allow for quicker access to the outlying regions. Thus, while there is some variability in the simulated response time distributions and 90th percentiles, the general trend suggests that moving the station along the highway on the western edge of the city will reduce



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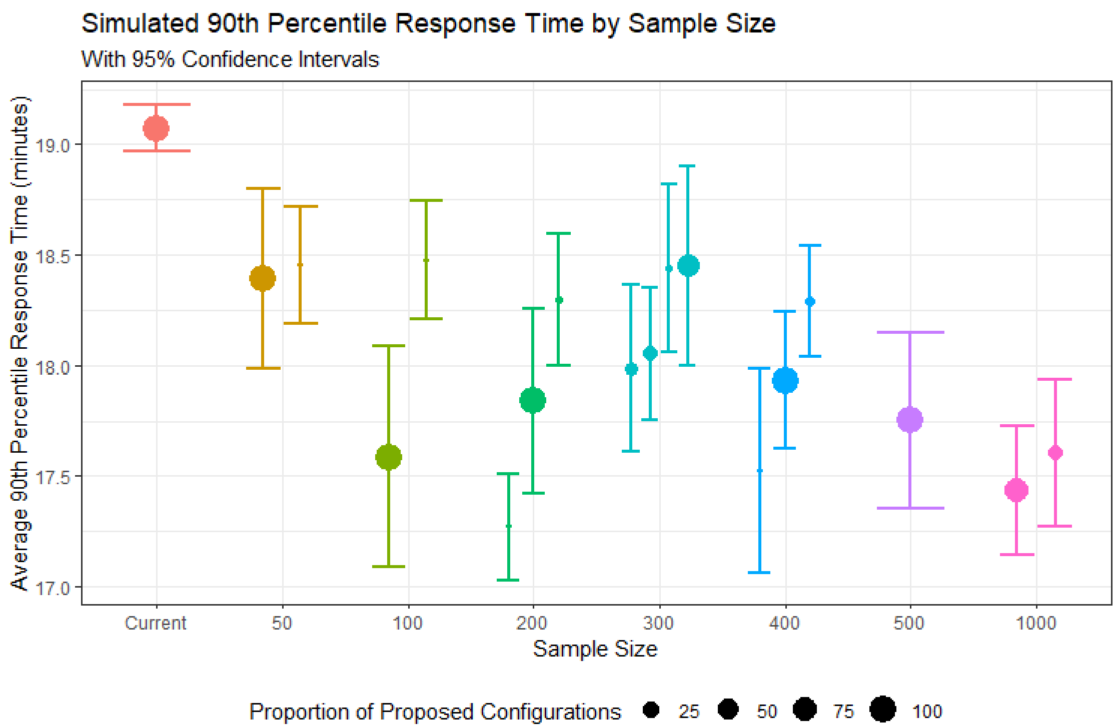
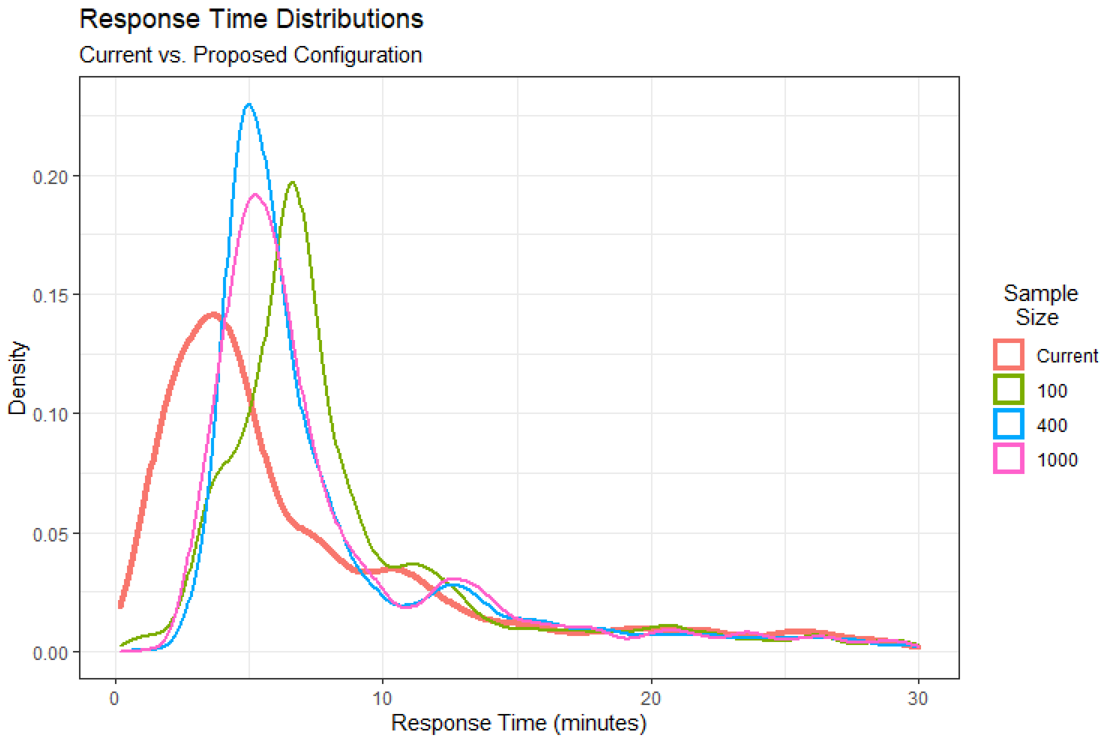
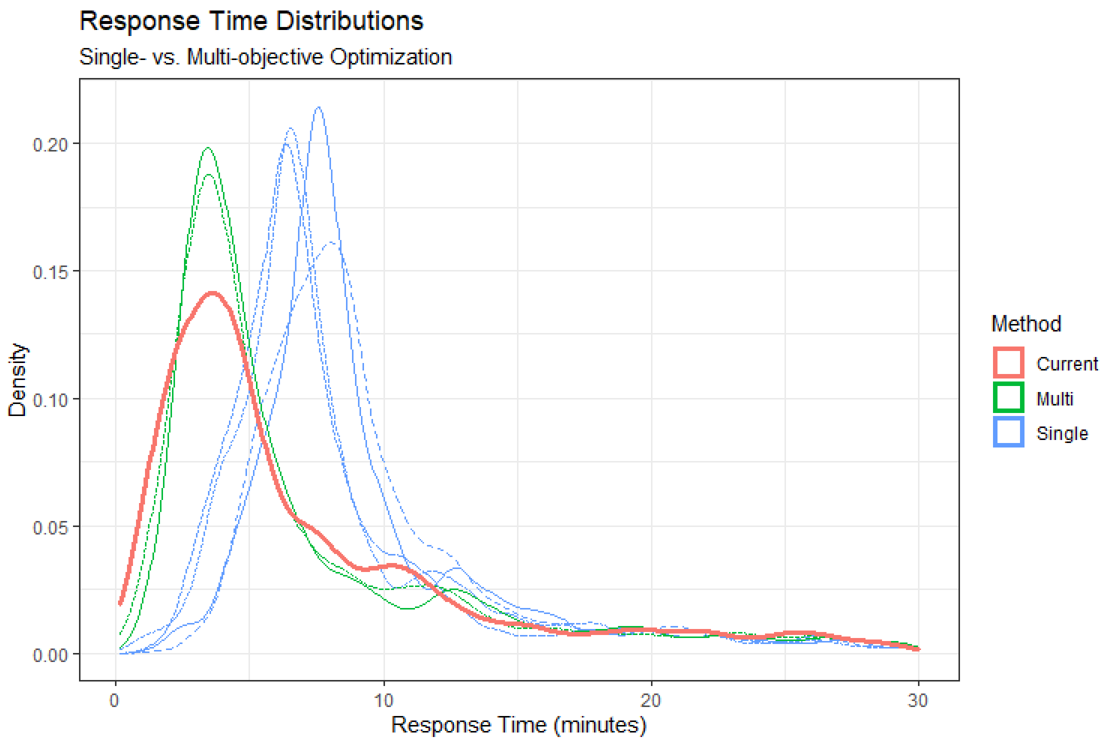


(b)

Figure 4: For the one-station setting, (a) metamodel fit by sample size with standard error bars; and (b) average DES simulated 90th percentile response times for proposed locations.



(a)



(b)

Figure 5: For the one-station setting, (a) distributions of response times for the current and proposed configurations; and (b) distributions of response times for proposed configurations based on the single- and multi-objective optimization discussed in Section 5.

the 90th percentile of the response times.

4.2. Two Station

Here we present results for the case in which two stations are considered. Weighted random samples of two-station configurations were taken according to the sample sizes specified in Table 3, and these samples were used to fit and optimize metamodels. Additional details are provided in Section 9 of the Supplementary Material.

Figure 7a again shows the goodness of fit of the models by sample size using the cross-validated R^2 of the selected random forest metamodel along with standard error bars. We see that though the sample sizes are necessarily larger than the one-station version, we can still achieve an informative metamodel.

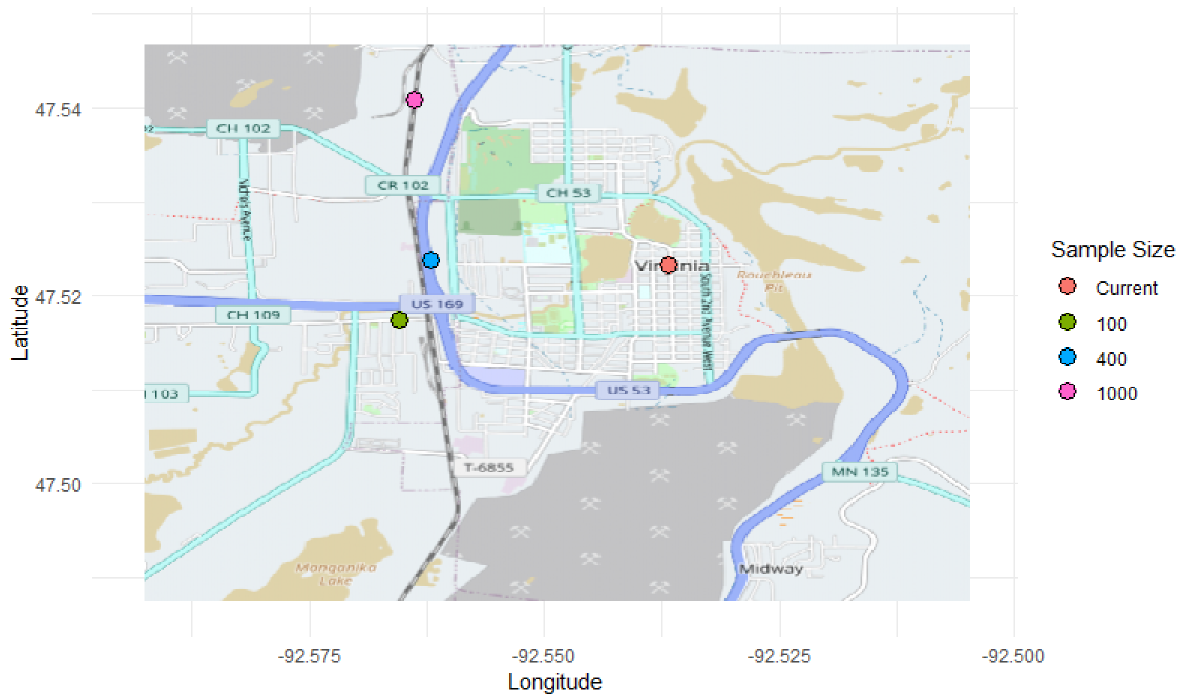
The results of the optimization are not as clean here, with more diversity of locally optimal solutions for each sample size. With the exception of a few outliers, the DES suggests that the metamodel-proposed solutions improve over the current configuration (Figure 7b). The best configuration was found by two optimization procedures on the metamodel built on a sample of 5,000 configurations, with a average 90th percentile of 13.50 minutes.

Figure 8 shows the distributions of simulated response times for the best configurations indicated by the metamodel with sample sizes of 1,000, 3,000, and 5,000. As with the one-station setting, while these proposed configurations decrease the 90th percentile, they increase the lower quantiles of response times. The peak of the distributions for each proposed configuration represented is shifted to the right of the current configuration's peak, signifying longer response times for many of the calls that are within three minutes of a response under the current system.

To understand this trend, Figure 9 shows the geographic locations of the most commonly proposed configuration for each sample size. The current configuration is shown in red, with locations in the center of Virginia and Eveleth. Each proposed configuration consisted of one station in the northern area to the west of Virginia and one station in the southern region to the southeast of Eveleth. All proposed configurations have locations closer to the highway than the current configuration, with the distances between stations slightly farther than the current distance. Similar to the one-station case, this general trend suggests that moving the stations closer to the highway would reduce the 90th percentile of the response times.

4.3. Five and Twelve Stations

Here, we provide briefer results for the five- and twelve-station versions. Supporting details are provided in Section 10 of the Supplementary Material. Overall, the metamodels for these two scenarios needed much more data than the one- and two-station systems, while at the same time requiring much larger computational resources to simulate and optimize. These were challenging scenarios that stretched our methodology and computational infrastructure, and consequently, the results are less encouraging. Still, it is important to demonstrate both where these methods excel and where they require more development.



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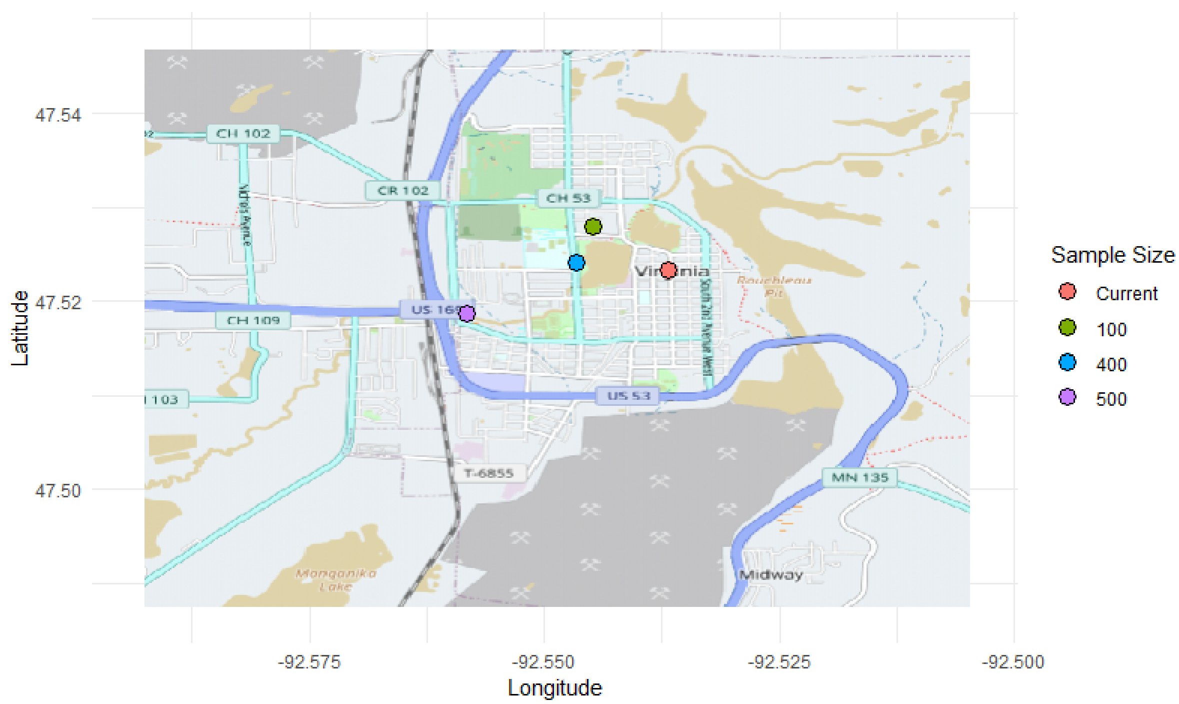
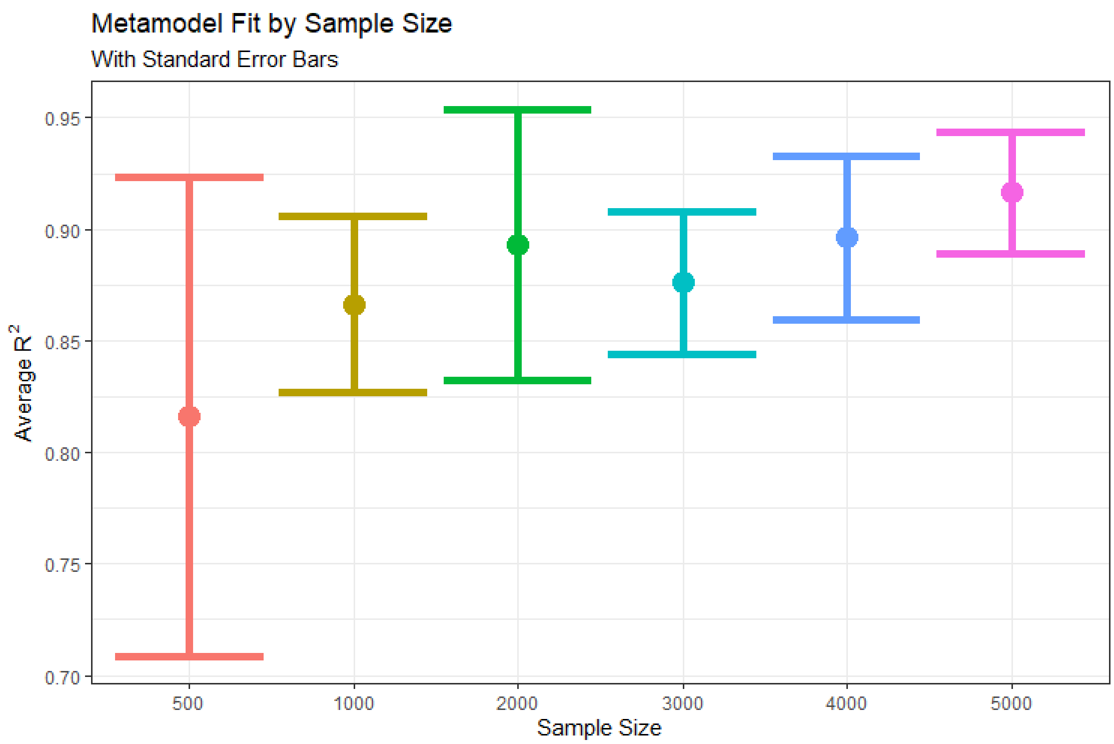


(b)

Figure 6: For the one-station setting, (a) location of proposed configurations in relation to Virginia; and (b) location of proposed configurations from the the multiobjective median-90th percentile optimization described in the Discussion. See Section 5.



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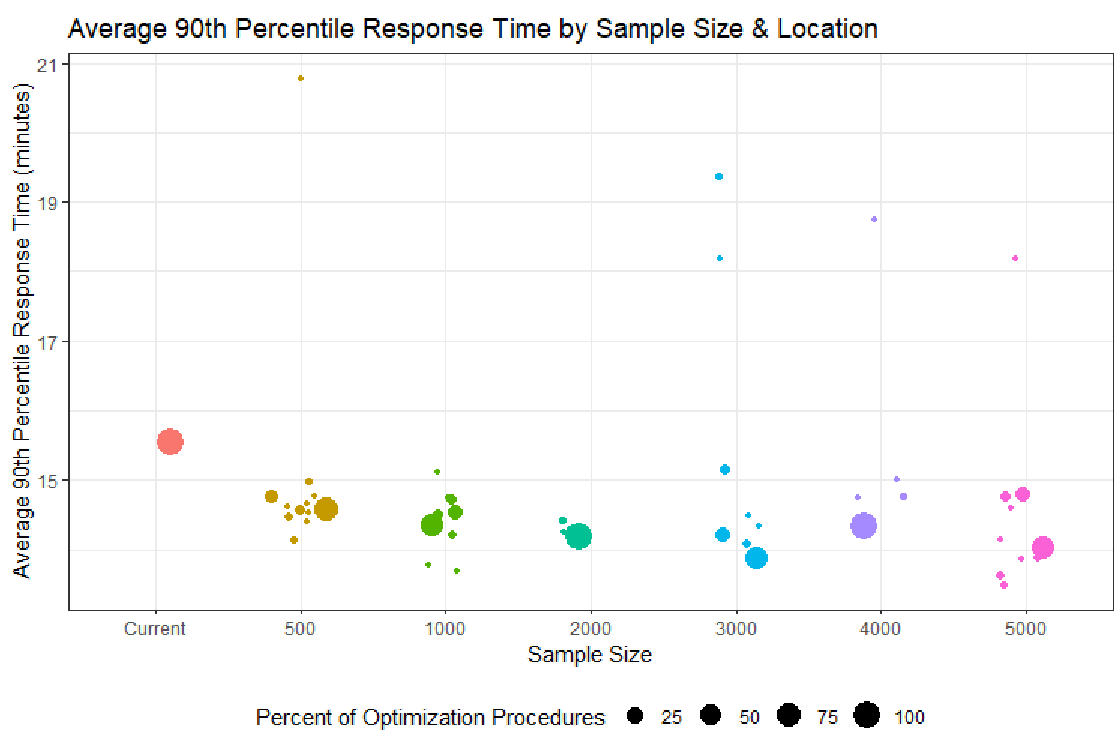


(b)

Figure 7: For the two-station setting, (a) metamodel fit by sample size with standard error bars; and (b) average DES simulated 90th percentile response times for proposed locations.

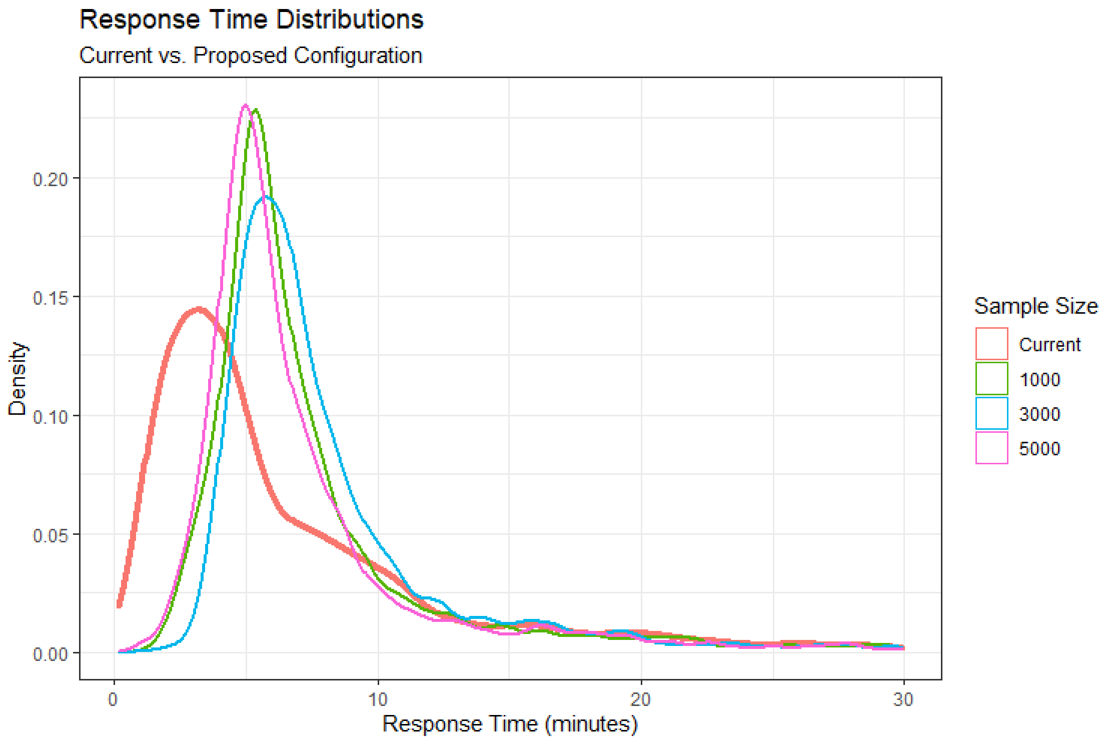


Figure 8: Distributions of response times for the current and proposed configurations, for the two-station setting.

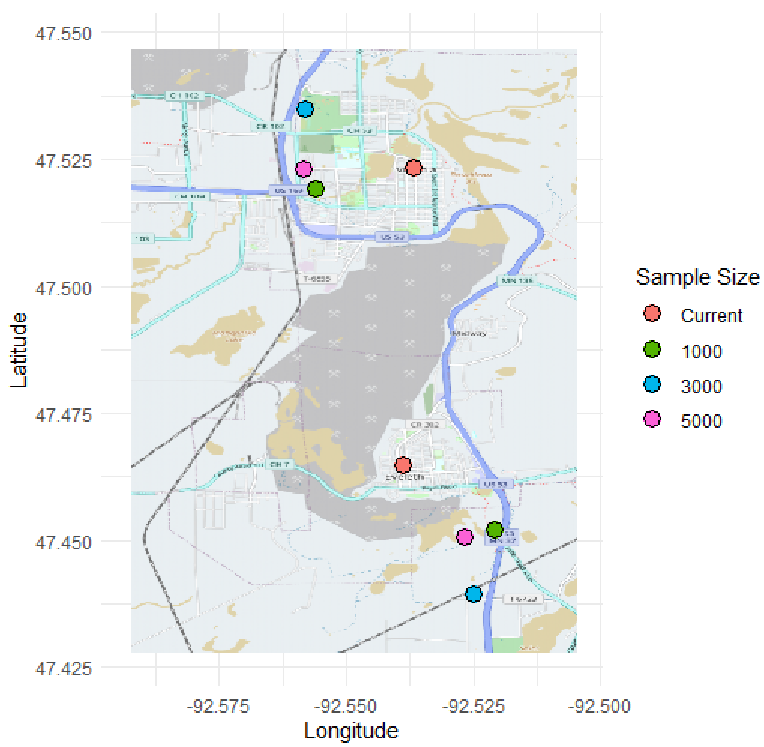
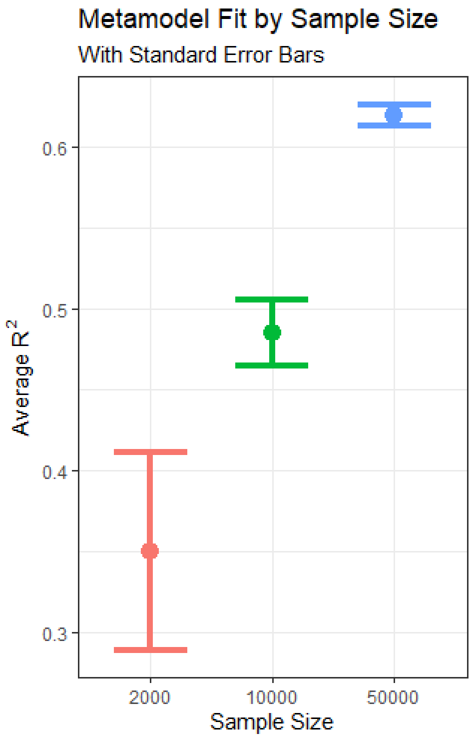
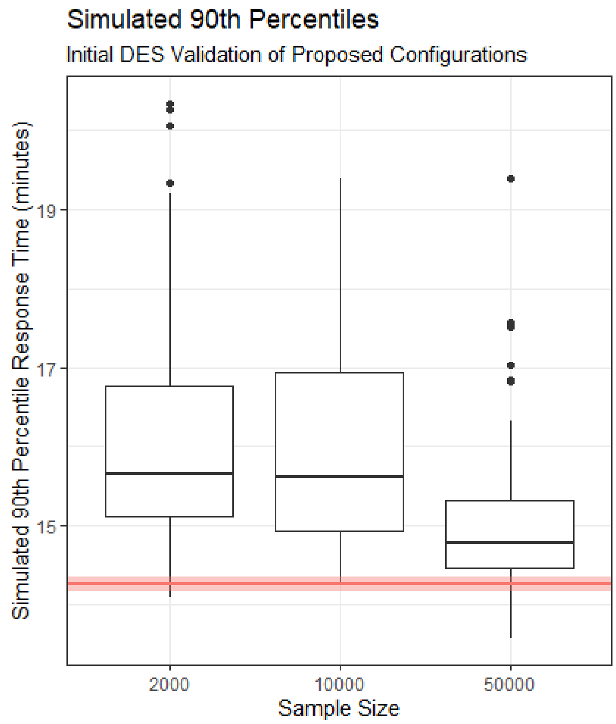


Figure 9: Location of proposed configurations in relation to St. Louis County, for the two-station setting.



(a)



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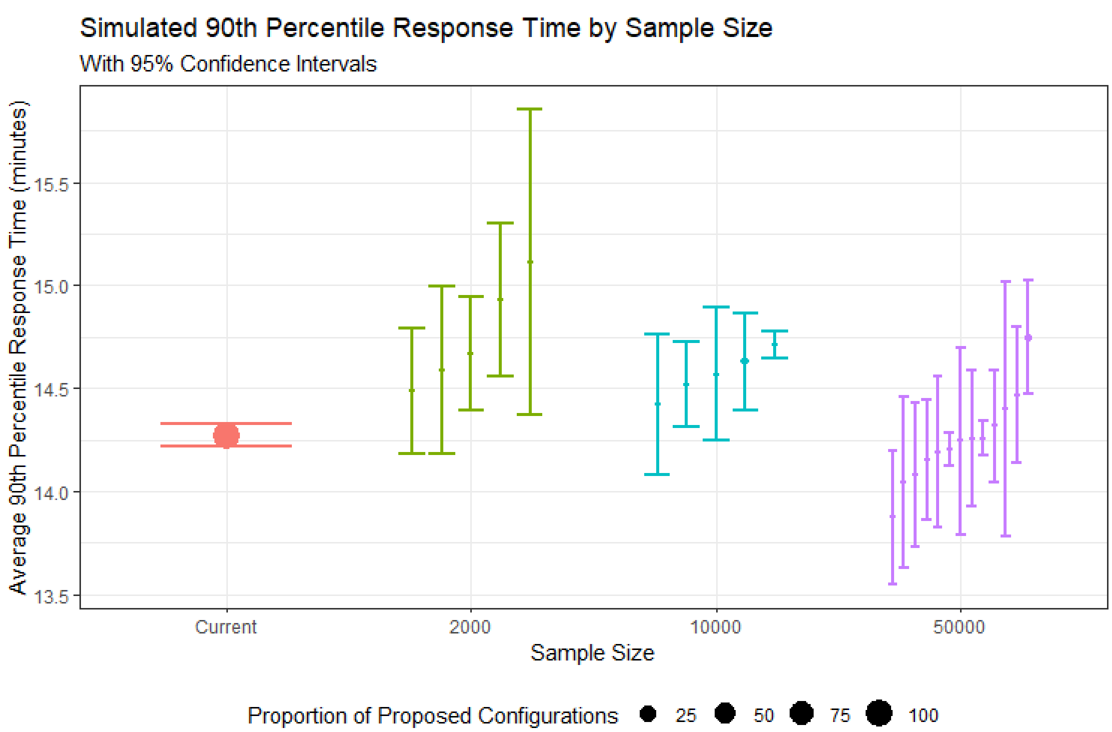


(c)

Figure 10: For the five-station setting, (a) metamodel fit by sample size, with standard error bars; (b) boxplot of single DES validation of 100 proposed configurations produced by optimizing the metamodels; and (c) an evaluation of several promising solutions on the DES. The red line in (b) is the DES performance of the current configuration, with 95% confidence band.

Figure 10a shows that the quality of fit improves as a function of sample size, and there is no indication that the rate of improvement is slowing down for the five-station setting. This suggests that given more computational resources, larger sample sizes would result in higher average R^2 values.

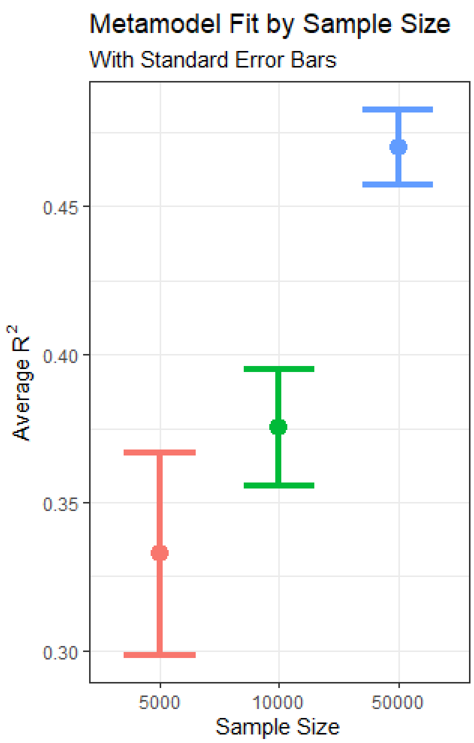
Figure 10b shows a boxplot of simulated 90th percentile response times for the initial validation of each proposed configuration by sample size, along with a horizontal line representing the average simulated 90th percentile under the current configuration. It indicates that most of the solutions produced by optimizing the metamodels are, according to the DES, inferior to the current configuration; however, for the 50,000 case, there are a few solutions (represented by the lower whisker on the box-and-whisker plot) that offer a potential improvement. In fact, 13 configurations in the 50,000 case have simulated 90th percentiles below the upper 95% confidence bound of that of the current configuration. For configurations in each sample size that rivaled the 90th percentile of the current configuration, we ran the DES several additional times to compute an average. Figure 10c shows the current configuration in red, along with the average and 95% confidence intervals of the simulated 90th percentiles for each of these configurations. None of the proposed configurations based on samples of size 2,000 or 10,000 proved superior to the current configuration, while several configurations based on the sample size of 50,000 had lower average simulated 90th percentiles, though the difference was less than 30 seconds for each.

For the twelve-station case, the problem took substantially longer to simulate and optimize, while requiring even more data. As with the five-station case, due to the amount of time needed to run the DES and generate data for the metamodels, only three different sizes of configuration samples were generated using the constrained, weighted method. Figure 11a shows the results; note that even with a sample size of 50,000, the cross-validated R^2 is not even 50%. We then used PSO to optimize each metamodel, and as with the five-station case, the complexity of the problem resulted in a majority of the procedures converging to different configurations.

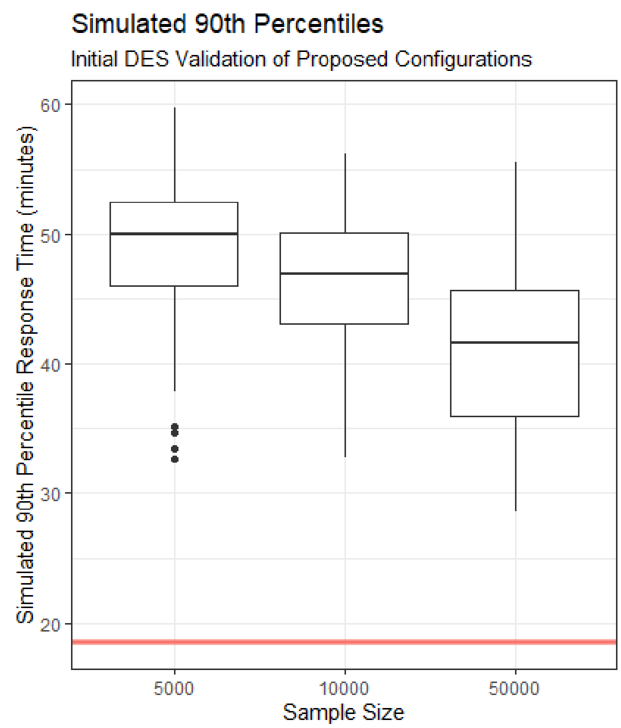
As with the five-station setting, increasing the sample size did result in lower overall distributions of 90th percentiles for the 100 configurations, but in this case nearly all times were still greater than 30 minutes and none approached the current average of 18.53 minutes (Figure 11b). Clearly, the metamodels failed to adequately capture the complexities of the system. More data is needed to train the metamodels in this case.

5. Discussion

Because detailed discrete event simulators for large rural-urban EMS systems are computationally demanding, it is difficult to optimize them directly, and even more challenging when including randomness in the simulation. To address this problem, we use metamodels as a surrogate for the DES. In particular, we studied how much data these surrogates required as the complexity of the system increased. We found that in order to model the 90th percentile response time of a single station, we only needed 200 to 400 data points to obtain an accurate random forest metamodel. This metamodel, when optimized, produced configurations



(a)



(b)

Figure 11: For the twelve-station setting, (a) metamodel fit by sample size, with standard error bars; and (b) boxplot of single DES validation of 100 proposed configurations produced by optimizing the metamodels. The red line in (b) is the DES performance of the current configuration, with 95% confidence band.

that improved upon the current configuration by over a minute, according to the DES. For the two-station setting, around 3,000 data points were required, and resulting optimized configurations suggested improvements of at least 30 seconds. For the five- and twelve-station settings, however, even 50,000 data points was inadequate to reliably optimize the system. For the five-station scenario, a few configurations were suggested that could offer small improvements, but for the twelve-station system the complexity overwhelmed the metamodel. Thus, while using a constrained, weighted sampling technique to generate data in intuitively promising regions decreased the number of DES runs needed, the metamodeling approach still requires a significant amount of time and data for complicated settings. We expect these findings can be generalized to similar rural-uban EMS systems, with the quality of the metamodel—as measured, say, by out-of-sample R^2—serving as an indicator of how effective the optimization will be.

Interestingly, we found that the best, most reliable metamodel fits were not necessarily needed to find improved locations, and many models with poorer fits also proposed configurations with good average 90th percentile response times. This suggests an important point: metamodeling may not produce a single, globally optimal solution, but is effective at collecting a set of promising solutions that can then be more carefully evaluated and analyzed using a simulator and domain knowledge. This collection happens naturally when an optimization heuristic such as particle swarm optimization is used and produces a number of solutions based on optimization runs from different initializations. This set of solutions can then reveal what is in common among improved configurations. For instance, for the one- and two-station cases, we see that many of the suggested solutions moved the stations closer to the highway (see Figures 6a and 9). This makes intuitive sense because we are optimizing the 90th percentile of the response time distribution; we are willing to allow an increased median response in order to reduce the right tail response by improving highway access (see Figures 5a and 8).

A simple question then presents itself: can we achieve the benefits of a reduced 90th percentile while also retaining a median response that is close to the current level? To briefly investigate this, we used a simple multiobjective optimization approach to simultaneously minimize the 90th percentile and median response times for the one-station case. In this optimization, we created a new objective function as a linear combination of 90th percentile and median, where both objectives were equally weighted. The analysis was done for sample sizes of 100, 200, 300, 400, and 500. Due to the increased stability around the median quantile, the R^2 for these metamodels were typically 5-10 percentages points higher than those modeling the 90th percentile. After the optimization and validation steps, we found that most proposed locations decreased the 90th percentile by approximately 20 to 60 seconds without drastically affecting the median. Figure 5b shows the response time distributions for the current station location in red, proposed locations based on the single objective analysis in blue, and the proposed locations based on the multiobjective analysis in green using a sample size of 300; the multiple density curves within each analysis type represent the different locations suggested by the 100 different optimization procedures. Geographically, many of the proposed locations under the multiobjective optimization procedures were on the western edge of the city of Virginia, but not as close to the highways as seen in Figure 6a, which balances access to the outlying areas with proximity to the high density area. These proposed locations are seen in Figure 6b, color coded by the sample size used to build the underlying model.

Based on this analysis, the discrete event simulators constructed to model EMS systems in St. Louis County, Minnesota appear to be reasonable approximations of the system's responses to medical calls. The use of randomness in each aspect of the simulator, guided by historic call data, strengthens the reliability of the DES by ensuring that anomalous calls do not drastically alter a given station configuration's responses. However, these simulators are overly simplified representations of the true system and do not account for the many complexities. For instance, all calls are assumed to have the same priority and only medical calls were considered, even though resources at these stations are shared between medical and fire emergencies. In addition, it is assumed that only one vehicle is needed for each call, and this responding vehicle is an ambulance that always starts from the station, not the hospital, a previous call location, or any location in between. Thus, while this DES can be used to find improved configurations, a more complex simulator could provide more insight.

There are many areas of future work possible to continue this analysis. To start, developing a faster simulator would allow for a fuller understanding of the amount of data required for the more complex cases with many stations. Additionally, rather than building a single metamodel on many runs of the DES, exploring model averaging techniques for many metamodels built on smaller samples could provide valuable insight with fewer runs. Alternatively, since an EMS system is unlikely to have the capacity to move all stations in practice, an alternative approach that focuses on either moving a subset of the stations or removing certain stations, while keeping the others in place, could be considered. Another approach to reduce the amount of data necessary to fit quality metamodels would be to continue exploring alternative sampling strategies. Techniques incorporating spatial densities and inverse-distance weights could be further explored, in addition to adaptive sampling, also known as active learning, which has been shown in the metamodeling literature to produce small yet informative samples [29, 31]. Along with the continuous optimization of station locations, incorporating station resources such as vehicles or personnel in the optimization would allow for more thorough solutions; for instance, perhaps combining the resources of two stations into a single station could reduce the response times while also reducing cost. Finally, performing a full multiobjective optimization analysis with Pareto fronts is a promising area of future research, as discussed above. Invariably, EMS departments care about more than one measure, and a multiobjective approach will provide decision-makers with a more realistic menu of options to improve their systems.

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